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Crest Factor Reduction of an OFDM/WiMAX Network

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Abstract: African countries lag behind the rest of the world in their use of Information and Communication Technologies (ICTs). To reduce the digital divide quickly and cost-effectively, wireless networks are considered. WiMAX (Worldwide Interoperability for Microwave Access) is a wireless broadband access technology that uses Orthogonal Frequency Division Multiplexing (OFDM) which is a multicarrier modulation scheme. OFDM presents a problem of a high crest factor or Peak to Average Power Ratio (PAPR). To circumvent this problem either High Power Amplifiers (HPAs) with large dynamic range or PAPR reduction techniques are used. The former scheme increases cost of the system while the latter introduces redundancy or distortion. A novel PAPR reduction scheme is presented. It is a combination of the ideas of Tone Reservation and Selected Mapping. The advantage of this scheme is that it has a lower complexity. It is simulated for a WiMAX system.

Keywords: Crest Factor, OFDM, PAPR, SLM, TR, WiMAX, Selected TR

1. Introduction

There is growing demand for broadband services world over. To achieve higher data rates to support broadband applications in a multipath environment, more effective modulation schemes in the form of multicarrier schemes are adopted. They present challenges of their own. This paper considers the high crest factor/PAPR of OFDM systems [1], [2]. The effects of a high PAPR are: the low power efficiency of the High Power Amplifier (HPA) and signal distortion caused by operation in the nonlinear region of the HPA. Low power efficiency implies that the HPA must have a large dynamic range thus increasing cost of the system. Signal distortion induces a degradation of the BER. PAPR reduction techniques are used to alleviate the extremely high back offs and costly amplifiers [2].

A number of techniques have been proposed to reduce the PAPR of multicarrier modulation schemes. They are broadly categorized into two: deterministic and probabilistic techniques [3]. Deterministic methods limit the crest factor of the OFDM signals below a threshold level. Clipping and block coding belong to this category. Probabilistic methods statistically improve the characteristic of the crest factor distribution of the OFDM signals without signal distortion. Tone Reservation (TR), Selected mapping (SLM) and partial transmit sequence (PTS) are included in this category. The various approaches are quite different from each other and impose different constraints.

This paper provides a short review of TR, SLM and selected TR. It then presents the novel scheme which is a modification of selected TR. The target audience includes: academicians, policy makers, telecommunications operators and manufacturers of telecommunication equipment.

2. Objectives

2.1 Main Objective

The main objective of this study is to reduce the PAPR of OFDM for WiMAX:

1. To prevent clipping of the transmit signal when the HPA operates in the nonlinear region
2. To reduce cost of WiMAX transmitters thus allowing their fast deployment especially in developing countries.

2.2 Specific Objectives

The specific objectives to achieve the PAPR reduction are:

1. To review literature on PAPR reduction techniques
2. To identify techniques suitable for WiMAX
3. To develop a better technique from those identified in the second specific objective which combines their advantages and minimizes on their limitations.
4. To simulate the use of the developed technique for WiMAX
5. To evaluate the performance of the developed technique

3. Methodology

In order to achieve the research objectives, this research used simulations as the primary methodology for the experimental findings of the improved crest factor technique. This section gives details of all that was involved in this research.

3.1 Research Design

This research was quantitative with a narrow focus on WiMAX networks or IEEE 802.16D standard with a physical layer based on OFDM. An improved PAPR / crest factor reduction technique was developed by combining the ideas of selected mapping and tone reservation which are popular PAPR reduction techniques.

3.2 Variables

The amount of PAPR reduction is determined by the peak reduction tones (additive signals) used therefore the variables that were considered for this study include:

- Number of PRTs (Peak Reduction Tones)
- Power of PRTs
- Number of mappings for SLM
- PAPR Threshold

3.3 *Data Collection Methods*

MATLAB was used to simulate the developed technique for the WiMAX network. YALMIP, a free optimization toolbox was used to obtain the additive signal that was used to reduce the PAPR. This tool is available on the internet and can be used with MATLAB. It employs a number of solvers including: SEDUMI (which is freely available on the internet), SDPT3, MOSEK and QUADPROG (which is integrated into MATLAB).

4. Technology Description

4.1 *PAPR Problem in OFDM Systems*

The OFDM transmit signal is a sum of the signals on all the orthogonal sub-carriers. In the discrete time domain, an OFDM signal, a_t, of carriers is expressed in Equation 1.

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left(A_n \, e^{j2\pi \frac{n}{N} t} \right), 0 \le t \le N-1 \quad (1)$$

In Equation 1 $A = [A_0 A_1 \dots A_{N-1}]$ is an input symbol sequence, N is the number of carriers, n is a running index and t stands for a discrete time index. The PAPR of an OFDM signal, defined as the ratio of the maximum to the average power of the signal is given by Equation 2.

$$PAPR(a) \triangleq \frac{\max_{0 \le t \le N-1} |a_t|^2}{E[|a_t|^2]} \quad (2)$$

In Equation 2, E[.] denotes the expected value, and $a = [a_0 a_1 \dots a_{N-1}]$ is the time domain input symbol sequence. PAPR is a well known measure of envelope / amplitude fluctuations and has become the cost function used to evaluate and design multicarrier systems such as OFDM systems [4]. As PAPR is a random variable thus it is evaluated in terms of its Complementary Cumulative Distribution Function (CCDF), that is, the probability that the PAPR exceeds a given threshold (γ^2) as shown in Equation 3.

$$CCDF(PAPR(a)) = \Pr\,(PAPR(a) > \gamma^2) \quad (3)$$

The signal transmitted by the OFDM system is the superposition of all signals transmitted in the narrowband sub-channels. The transmit signal has then due to the central limit theorem a Gaussian distribution leading to high peak values compared to the average power. A system design not taking this into account will have a high clip rate: Each signal sample that is beyond the saturation limit of the power amplifier suffers either clipping to this limit value or other non-linear distortion, both creating additional bit errors in the receiver [1], [5], [6].

4.2 *PAPR Reduction Techniques*

Tone Reservation (TR) and Selected Mapping (SLM) are some of the popular PAPR reduction techniques. They are briefly described in this section.

4.2.1 *Tone Reservation*

Tone Reservation (TR) cancels out the peaks in the OFDM data signal using an additive signal. The signal addition is achieved in frequency domain as shown in Figure 1 where corrective carriers are added in between the useful data carriers. A block diagram of this PAPR reduction scheme is depicted in Figure 2. The OFDM signal is denoted by x, the corrective additive signal by c and the transmit signal as d which is a sum of x and c. The data signal is first converted to the time domain using the IDFT (Inverse Discrete Fourier

Transform) then it is fed into an algorithm which generates the additive signal. This scheme searches for an additive corrective signal, c, in order to have PAPR(x + c) < PAPR(x). Thus, the PAPR of the resulting signal is given in Equation 4.

$$PAPR = \frac{\max_{0 \le k \le N-1} |x_k + c_k|^2}{E[|x+c|^2]} \qquad (4)$$

The reserved tones, also referred to as Peak Reduction Tones (PRT), are selected from those that cannot be used for data transmission because of their low SNR (Signal to Noise Ratio) [8]. One way to find c is to use optimization approaches. Various schemes have been proposed to obtain the additive signal, c.

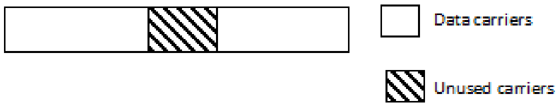


Figure 1: Unused Carriers are Used to Reduce PAPR in Tone Reservation

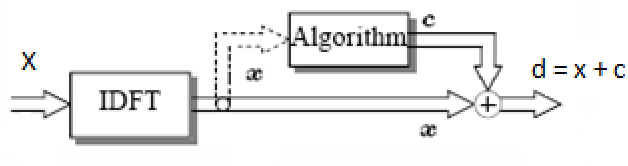


Figure 2: Block Diagram of TR [7]

The oldest technique is Tellado's Signal to Clipping Noise ratio (SCR) or gradient algorithm [9], [10], [11]. Other schemes have been developed that reduce on the complexity of Tellado's approach that include: active set method, Selective mapping of Partial Tones (SMOPT), One-Tone-One-Peak (OTOP) and one-by-one iteration. These techniques also improve on the performance of Tone reservation [12]. The information carrying signals and the additive signals can be distinguished at the receiver after Discrete Fourier Transform (DFT) hence there is no signal distortion. And therefore there is no need for transmission of side information as is the case in selected mapping.

4.2.2 Selected Mapping

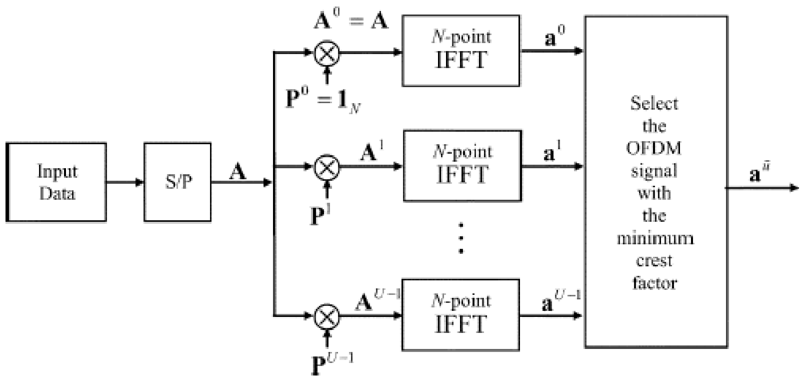


Figure 3: Block Diagram of the SLM OFDM Scheme [13]

Selected mapping (SLM) involves the adjustment of the phase of the sub-carrier (OFDM data) signals in the time domain in order to optimize the composite OFDM waveform that is eventually transmitted [14]. A set of alternative symbol sequences (i.e. $A^u = [A_0^u A_1^u \ldots A_{N-1}^u]$) is generated from a given input symbol sequence (i.e. $A = [A_0 A_1 \ldots A_{N-1}]$) which is multiplied by the phase sequences ($P^u = [P_0^u P_1^u \ldots P_{N-1}^u]$) as illustrated in Figure 3. U is the number of mappings; u is a running index which ranges from 1 to U (i.e. $1 \le u \le U$) and N is the number of subcarriers [15]. The procedure used by SLM as shown in Figure 3 is described as follows:

1. The input OFDM data signal is first fed into a serial to parallel converter (denoted as S/P in Figure 3).
2. Then each symbol (A) is fed into a multiplier whose other input is a phase sequence (P) usually a row of a chosen matrix such as the Hadamard matrix. The first phase sequence usually consists of ones (1s).
3. The output (A^u) of the multiplier is then converted into the time domain using an N-point IFFT (Inverse Fast Fourier Transform) block.
4. Finally the PAPR of each alternative symbol sequence (a^u) is calculated and compared with the others. Then the symbol with the minimum PAPR/crest factor in the set is selected for transmission.

The SLM technique effectively randomises the phase of each sub-carrier so that when sub-carriers are added together, the signals are less likely to be in phase with one another and the resultant OFDM waveform is flatter. The original OFDM symbol, A, is recovered from the received signal A^u by multiplying the conjugate of applied phase sequence/vector, P^u, to the received signal. Knowledge of the applied phase sequence is needed at the receiver. Information on the phase sequence used for the transmitted signal must be conveyed to the receiver in the SLM scheme. This is represented as an index symbol sequence which is augmented to the data symbol sequence. The index /side information transmitted to the receiver in SLM introduces redundancy and reduces power efficiency.

4.2.3 Selected Tone Reservation

In [7] selected tone reservation is proposed as a variant of TR. It is a combination of the ideas of TR and SLM. Instead of applying different mappings on the original OFDM frame, this scheme creates *U* multiple signal representations of the additive signal, c, and then adds them to the original data signal, x. In this way U alternative symbols are obtained from which the one with the lowest PAPR is selected for transmission. The huge benefit of this technique is that these *U* additive signals can a-priori be chosen and transformed into time domain. Thus only one IDFT has to be calculated per OFDM frame (instead of *U* calculations required for the SLM technique), whereby the number *U* of assessed signal candidates might be much larger than with SLM. Selected TR has a lower complexity than TR and does not require transmission of side information. However, it is significantly outperformed by both TR and SLM.

5. Developments

This section describes the novel scheme developed by the authors. The novel PAPR reduction scheme is based on selected tone reservation. The PAPR reduction is applied after the N point IFFT process as shown in the block diagram of the novel scheme in Figure 4. It uses a threshold PAPR to determine the symbols that should undergo selected TR processing. The PAPR threshold is denoted by γ_T . The algorithm used is as follows:

If PAPR(x) < PAPR Threshold; No PAPR reduction
Else {
Do PAPR reduction i.e. x_new = x + c }

The PAPR Threshold is a pre-determined value. The use of the threshold is based on the premise that reducing the PAPR which is way below the amplifier clipping level is of little gain. On the other hand, by using the threshold the computational complexity is reduced since unnecessary processing is eliminated. The additive signal, c, is determined a prior using SOCP (Second Order Constraint Program) [10].

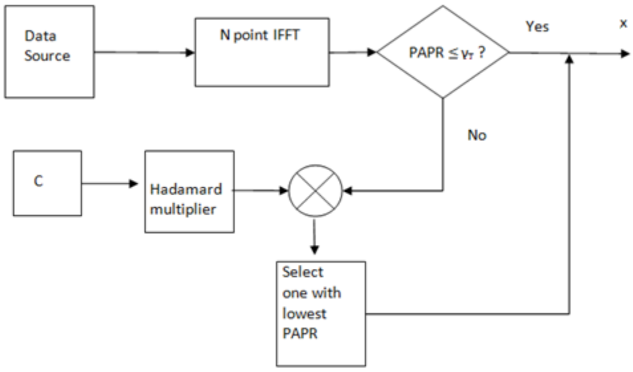


Figure 4: Block Diagram of Selected TR with a Threshold

The novel technique follows the steps described below:

1. Obtain the PAPR for each symbol, x
2. Then check whether the PAPR is larger than a pre-determined PAPR threshold, if it is proceed to step 3 otherwise the signal is unmodified
3. Generate multiple sequences of c by multiplying it with (U-1) randomly selected rows of a Hadamard matrix. The first sequence of c is always multiplied by one (1). A constant gain is sometimes used to boost the power of c in order to increase its effect.
4. Form U sequences of the data signal by adding to it the sequences of c i.e. ($d_i = x + c_i$). The running index, i, has the range $1 \le i \le U$
5. Lastly calculate the PAPR of the various sequences of the data signal and select the one with the lowest for transmission.

When the randomly selected row in step 3 above is one (1), then the processor selects the second row. The first row of a Hadamard matrix consists of ones (1) only.

6. Results

This section presents the results from simulations performed to evaluate the performance of the novel PAPR technique. The simulations are performed for the IEEE 802.16D standard. This standard specifies 256 sub-carriers out of which only 192 can be used for data transmission. The parameters for the OFDM/WiMAX system are given in Table 1. 10,000 input symbol sequences were randomly generated and are therefore independent and identically distributed (i.i.d.).

*Table 1: Simulation Parameters for IEEE 802.16*D

Symbol	Description	Value
N	FFT Size	256
x	Number of data tones	192
Dc	Dc tone	1
Pilot	Pilot tones	8
Unused tones	Not used for data transmission + dc	56
Modulation	Type of modulation	64 QAM
U	Number of SLM mappings	Power of 2
L	Oversampling factor	Either 1 or 4

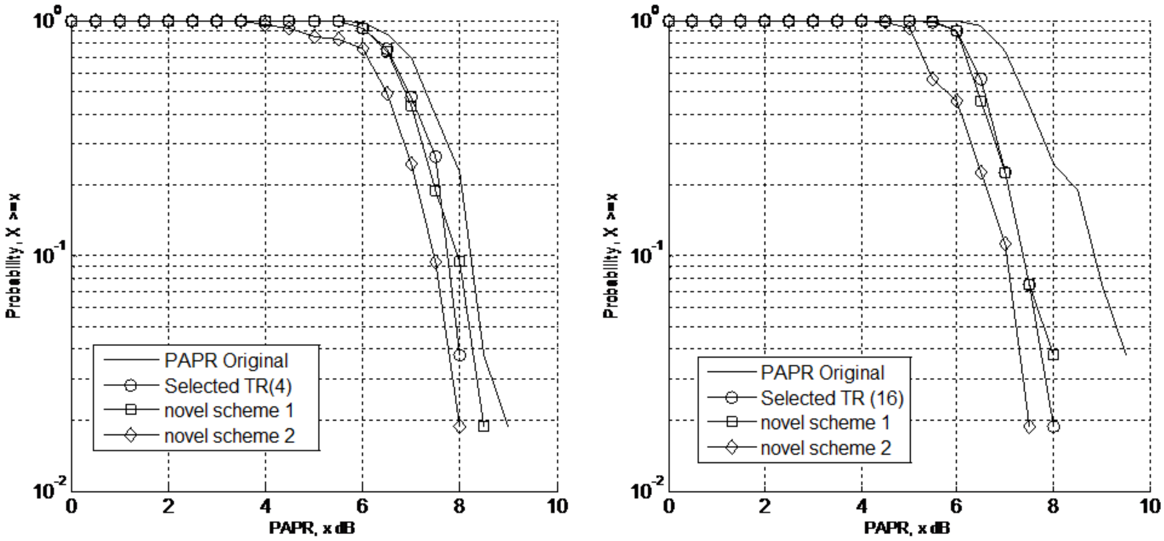
6.1 CCDF

There are two forms of the novel scheme: one which does not control peak re-growth and the other that controls peak re-growth. For the rest of this paper the former scheme is referred to as novel scheme 1 and the latter as novel scheme 2. The threshold PAPR is set at 6 dB. The number of PRTs is the same as that used for selected TR. 32 PRTs were used i.e. 12.5% of the total carriers were dedicated to PAPR reduction. This is the largest number of PRTs that can be used albeit there are 55 unused tones for the IEEE 802.16D standard. This is dictated by the SLM program which uses an (n x n) Hadamard matrix for which n is equal to the length of c i.e. the number of PRTs and it must be a power of 2. The data is not oversampled i.e. L = 1.

Figures 5a and 5b show the CCDF plots for the PAPR of the novel schemes for U = 4 and 16 mappings respectively. They are compared against the original selected TR scheme and the signal without PAPR reduction. The PAPR in decibels is on the abscissa while the probability of the PAPR being greater than a certain threshold is indicated on the ordinate. The plot for the unmodified signal is the line without markers while the circular markers are for the signal with the original selected TR, the square markers for novel scheme 1 and the diamond markers for novel scheme 2.

From both Figures 5a and 5b, for any probability level, Novel scheme 2 gives the lowest PAPR values. It achieves the largest PAPR reduction and therefore gives the best performance where PAPR reduction is the difference between PAPR of the unmodified signal and that with a PAPR reduction technique applied.

On comparing Figures 5a and 5b it is observed that the greater PAPR reduction is achieved for 16 mappings. Therefore increasing the number of mappings increases the PAPR reduction and hence the performance of the scheme.



Figures 5a and 5b: CCDF Plots for U = 4 and 16 Respectively

7. Business Benefits

PAPR is gaining more recognition in the telecommunications sector world over. The findings of this research can be used by regulators and policy makers of the telecommunications sector to design best practices for PAPR reduction. This paper presents a PAPR reduction technique that is less complex and hence would be cheaper to implement than conventional reduction techniques. By using PAPR reduction techniques the cost of transmitters can be reduced.

Through simulations it has been shown that the novel technique reduces the PAPR of an OFDM/WiMAX network. However it is doubtful as to whether this scheme will still reduce the PAPR as expected when applied on hardware.

8. Conclusions

The novel PAPR reduction scheme presented in this paper is a combination of the ideas of TR and SLM which are popular schemes. This scheme is less complex than TR and selected TR since it does not find an additive signal for every OFDM symbol. In addition it does not require transmission of side information unlike SLM and therefore does not introduce redundancy.

The power of the additive signal is a major factor in the performance of the novel scheme. On one hand, by increasing its power there is an improvement in PAPR reduction on the other hand the transmit power increases and the likelihood of spectral mask violation is high. Further work may look into finding an optimized additive signal with spectral mask constraints applied.

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